DOCUMENT RESUME

ED 154 421 CS 204 157

AUTHOR Sternberg, Robert J.
TITLE Toward a Unified Componential Theory of Human Reasoning. Technical Report No. 4.
INSTITUTION Yale Univ., New Haven, Conn. Dept. of Psychology.
SPONS AGENCY Office of Naval Research, Arlington, Va. Personnel and Training Research Programs Office.; Office of Naval Research, Washington, D.C. Psychological Sciences Div.
PUB DATE Apr 78
CONTRACT N0001478C0025
NOTE 75p.

EDRS PRICE MF-$0.83 HC-$3.50 Plus Postage.
DESCRIPTORS *Cognitive Processes; Cognitive Style; Complexity Level; Critical Thinking; *Deductive Methods; Educational Psychology; Higher Education; *Inductive Methods; *Logical Thinking; Models; *Problem Solving; *Theories

ABSTRACT

The unified theory described in this paper characterizes human reasoning as an information processing system with a hierarchical sequence of components and subtheories that account for performance on successively narrower tasks. Both deductive and inductive theories are subsumed in the unified componential theory, including transitive chain theory of transitive inference, "IMAJER" theory (analogical, serial, metaphorical reasoning), and theory of causal inference. The basic unit of this unified theory is the "component," an elementary information process that is executed to solve some class of reasoning problem. As each component and subtheory is described in the paper, available data supporting its inclusion within the unified theory are provided; the data appear to be consistent with the proposed hierarchical structure. Although these data do not account for all of the performance variance, neither do they support any alternative explanations; moreover, no alternative account attempts to explain as great a range of human reasoning as does the unified componential theory. (RL)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED154421

Toward a Unified Componential Theory of Human Reasoning

Robert J. Sternberg

Department of Psychology
Yale University
New Haven, Connecticut 06520

U S DEPARTMENT OF HEALTH, EDUCATION & WELFARE
NATIONAL INSTITUTE OF EDUCATION

THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL NATIONAL INSTITUTE OF EDUCATION POSITION OR POLICY

Technical Report No. 4
April, 1978

Approved for public release; distribution unlimited.
Reproduction in whole or in part is permitted for
any purpose of the United States Government.

This research was sponsored by the Personnel and Training Research Programs, Psychological Sciences Division, Office of Naval Research, under Contract No. N0001478C0025, Contract Authority Identification Number NR 150-412.

AS 20W 157

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: Technical Report No. 4	2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle): "Toward a Unified Componential Theory of Human Reasoning"	5. TYPE OF REPORT & PERIOD COVERED: Periodic Technical Report (1 Jan 78 - 30 Mar 78)
	6. PERFORMING ORG. REPORT NUMBER: Research Report No. 4-78
7. AUTHOR(s): Robert J. Sternberg	8. CONTRACT OR GRANT NUMBER(s): N0001478C0025
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Department of Psychology, Yale University, New Haven, Connecticut 06520	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 61153N; RR 042-04; RR 042-04-01; NR 150-412
11. CONTROLLING OFFICE NAME AND ADDRESS	12. REPORT DATE: 1 Apr 78
	13. NUMBER OF PAGES: 72
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report): Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

reasoning, component, deduction, induction, unified componential theory

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Human reasoning can be characterized in terms of a unified theory that comprises hierarchically nested subtheories, each of which accounts for performance on successively more narrow tasks. The basic unit of the unified theory is the component: It is claimed that a relatively small set of components can account for behavior in a wide range of reasoning tasks, and that individual components are general across the vertical range of the hierarchy. The components and the subtheories in which

they are involved are briefly described, and where available, data testing the subtheories are presented as well. The data collected to date are generally consistent with the hierarchical structure proposed in the article. Although none of the accounts of performance are "true" in the sense of accounting for all of the reliable variance in the data, there are no alternative subtheories that are superior to any of the present ones in accounting for the data, and there is no alternative account at all that attempts to explain the range of reasoning data explained by the unified componential theory.

Abstract

Human reasoning can be characterized in terms of a unified theory that comprises hierarchically nested subtheories, each of which accounts for performance on successively more narrow tasks. The basic unit of the unified theory is the component: It is claimed that a relatively small set of components can account for behavior in a wide range of reasoning tasks, and that individual components are general across the vertical range of the hierarchy. The components and the subtheories in which they are involved are briefly described, and where available, data testing the subtheories are presented as well. The data collected to date are generally consistent with the hierarchical structure proposed in the article. Although none of the accounts of performance are "true" in the sense of accounting for all of the reliable variance in the data, there are no alternative subtheories that are superior to any of the present ones in accounting for the data, and there is no alternative account at all that attempts to explain the range of reasoning data explained by the unified componential theory.

Toward a Unified Componential Theory of Human Reasoning

During the past several years, I have been devoting the major portion of my research effort toward the development and testing of a "unified componential" theory of human reasoning. In order to understand the theory, it is necessary first to understand why the theory is "unified" and why it is "componential."

The proposed theory is "unified" because it attempts to explain within a single theoretical framework human information processing in a wide variety of deductive and inductive reasoning tasks. The unified theory comprises hierarchically nested subtheories accounting for performance on successively more narrow classes of tasks. The hierarchical structure of the theory is depicted by the tree-diagram in Figure 1. Corresponding to each node in the

---

Insert Figure 1 about here

---

hierarchy is a theory or subtheory of human reasoning, and a class or subclass of tasks to which the theory applies. Theories at each level of the hierarchy include as special cases all subtheories nested beneath them.

At the top of the hierarchy is the unified theory. Under the unified theory are two subtheories, one of deductive reasoning and one of inductive reasoning. In general, the theory of deduction applies to tasks in which there is a deductively certain solution, whereas the theory of induction applies to tasks in which there is no deductively certain (logically valid) solution, but there is an inductively probable one.

Each of these subtheories can again be split into two subtheories. In the case of the subtheory of deduction, the two further subtheories are ones of syllogistic reasoning and of transitive inference. The theory of syllogistic reasoning deals with class inclusion (categorical) and conditional

relations. The theory of transitive inference deals with transitive (linear ordering) relations. In the case of the subtheory of induction, the two subtheories are one of analogical, classificational, serial, topological, and metaphorical reasoning, and one of causal inference.

At the lowest level of the hierarchy are specific information-processing models that describe in detail the sequencing of components used in the solution of specific types of problems. Each model is expressed in terms of a flow chart that characterizes the course of information processing from the time the problem is first perceived until the time the individual makes a response.

Although the various subtheories differ in their level of generality and particular contents, the structure of each subtheory (and of the unified theory) is the same. The unified theory and subtheories each specify (a) the components of response time and response choice, (b) the representation(s) upon which these components act, (c) parameter estimates corresponding to the durations, difficulties, or probabilities of execution of these components, (d) strategies (rules) for combining the components, and (e) theoretically-based relations of the components to previously established, relevant psychological constructs (such as "reasoning" as measured by standardized tests of mental ability). Specification of each of these items requires a given theory or subtheory to be detailed and complete in its account of human information processing. At the heart of this account is the information-processing component.

The proposed theory is "componential" because the basic unit of information processing in the theory is the component: an elementary information process that is executed in the solution of some class of reasoning problems. The components of information processing pertinent to subtheories lower in

the theoretical hierarchy are pertinent as well to the higher-order subtheories under which the lower-order subtheories are nested.

The components of human reasoning are explanatory as well as descriptive constructs. They are the sources not only of communalities in the performance of multiple subjects in reasoning tasks, but also of individual differences in performance (see Sternberg, 1977b, Chapter 4). General, group, and specific factors obtained in factor analyses of ability tests, for example, can be accounted for in terms of distributions of components across tasks: A general factor arises when one or more components are common to all tasks; a group factor arises when one or more components are common to several tasks; a specific factor arises when one or more components are specific to a single task. From the standpoint of the componential approach to human abilities, therefore, components are the elementary units of analysis. Factors are merely constellations of these components that arise as a function of the particular mixture of components required for solution of a particular battery of items or tests subjected to a factor analysis (Sternberg, 1977b).

Most, if not all, components can be split indefinitely into successively smaller subcomponents. The level of division that is considered "elementary" for a given purpose is one of convenience, with convenience being determined among other things by (a) homogeneity of level of division of components within a single subtheory at a given level of the hierarchy, (b) generality of a component across tasks within a given node and at various levels of the hierarchy, and (c) univocal correlations of a component with scores on orthogonal mental ability tests. This last criterion requires that the parameter estimate corresponding to the duration, difficulty, or probability of component execution should be highly correlated with tests

measuring one kind of ability but only poorly correlated with tests measuring other kinds of abilities.

With these introductory remarks completed, it is possible to turn to the main body of the article, which will be devoted to a description of the contents of each node of the hierarchy. In describing these contents, it will be necessary to work through the hierarchy from the bottom up. Thus, the account of reasoning will proceed from the specific to the general. Only in this way can the reader see how very specific subtheories of very specific classes of tasks can be integrated to form successively more general subtheories of more general classes of tasks.

Where possible, the description of each node in the hierarchy will consist of (a) an example of the type of problem task that belongs at the given node, (b) a brief explication of the theory as it applies to that task, (c) a presentation of experimental data that have been collected in an attempt to test the theory. Neither the process of theory formulation nor that of theory testing is yet complete. Thus, it will be possible at some nodes only to fill in a description of the task to which the theory has yet to be applied. Nevertheless, theory construction and validation are far enough along to give the reader a good idea of the forms the final theory and data will take.

Consider now the contents (task description, theory description, data) of the nodes in the hierarchy, starting at the lower left with IA1.

COMPONENTIAL THEORY OF DEDUCTION

Transitive-Chain Theory of Syllogistic Reasoning

Models for Two Quantified Premises

Nature of Tasks. In syllogisms with two quantified premises, subjects are presented with two quantified premises describing relations between classes or events, and four possible conclusions that might follow logically from the premises. Each premise expresses a relation between two sets of objects or two events, one of which overlaps between the premises. The conclusion expresses a relation between the two nonoverlapping sets of objects or events. The subject's task is to select the preferred conclusion, or to indicate that none of the conclusions follow from the premises.

Syllogisms may be expressed in either categorical or conditional form. Examples of each type are the following:

No B are C.	If B occurs, then C does not occur.
Some A are B.	If A occurs, then B sometimes occurs.
All A are C.	If A occurs, then C occurs.
Some A are C.	If A occurs, then C sometimes occurs.
No A are C.	If A occurs, then C does not occur.
Some A are not C.	If A occurs, then C sometimes does not occur.
Indeterminate.	Indeterminate.

Subjects find problems of these kinds quite difficult. For example, mean response time for the categorical syllogisms was 39 seconds with a standard deviation of 8 seconds. What kinds of processes take place during the solution of these difficult problems?

Information-processing model. According to the proposed information-processing model (Guyote & Sternberg, Note 1), solution of syllogisms such as

the ones exemplified above takes place in four sequential stages: encoding, combination, comparison, and response. A list of the information-processing components hypothesized to be involved in each stage is shown in Table 1.

Insert Table 1 about here

During encoding, the subject forms a mental representation of each of the set relations that can be used to describe the verbal relation between the two terms in each premise. Although encoding of the possible set relations is assumed to be complete and correct, encoding of certain set relations is theorized to be easier and therefore more rapid than encoding of other set relations. Specifically, the equivalence relation (identical sets such as widows and women whose husbands have died) is assumed to be encoded more easily than nonequivalent but symmetrical relations (overlapping sets such as women and professionals; disjoint sets such as women and men), which in turn are assumed to be encoded more easily than asymmetrical relations (set-superset relations such as women and human beings; set-subset relations such as human beings and women). A symmetrical relation is defined here as one in which the relation of A to B is the same as the relation of B to A. In the example, the subject might first encode the symmetrical disjoint relation between B and C expressed by the first premise (No B are C). Since only one set relation can be used to describe the given verbal relation, encoding of that premise is complete. But there are four possible set relations that can be used to describe the verbal relation between the two terms of the second premise (Some A are B). The subject might first encode the equivalence set relation (A and B identical), then the symmetrical set relation (A and B overlapping), and finally the two asymmetrical set relations (A subset of B, A superset of B).

During combination, the subject integrates pairs of set relations encoded for each of the premises, combining A-B and B-C relations to form A-C relations. Certain pairs of set relations are theorized to be more easily combined than others. In particular, pairs of equivalence set relations are theorized to be easiest to combine, followed by an equivalence with a nonequivalence relation, followed by a pair of nonequivalent symmetrical relations, followed by an asymmetrical relation with any other kind of relation. In the example, the subject might first combine the symmetrical (B disjoint with C) relation with the equivalence (A identical to B) relation, then the pair of symmetrical relations (B disjoint with C--A overlapping with B), and finally the symmetrical relation with the two asymmetrical ones (B disjoint with C--A subset of B, A superset of B).

During comparison, the subject compares his or her mental representation(s) of the combined A-C set relation(s) to the verbal labels presented as conclusions. If one of the labels appropriately describes the mental representation, the subject is able to select a deductively valid conclusion. If not, the subject is theorized to check the operations of the combination stage for one or more possible errors, and to select the "Indeterminate" answer option if no errors are found.

During response, the subject communicates to the experimenter in some way his or her choice of an answer option.

In this and other information-processing models, each component of information-processing is assumed to be a real-time operation. Operations within stages, like the stages themselves, are assumed to be sequential, so that total processing time is proposed to equal the sum of the component times. The basic dependent variable for this and other information-processing models is thus response time, which is predicted from independent variables determined by the structural

properties of the syllogism or other problem type. Note, however, that this kind of model does not predict the distribution of response choices across the answer options presented to the subjects. In order to predict this distribution, a complementary but compatible model of response choice is needed.

Response-choice model. The components of the response-choice model, like those of the information-processing model, are organized according to the stage of information processing in which they have their effect. The components are listed in Table 2. Note that there are no components listed

---

Insert Table 2 about here

---

for either the encoding or the response stage, because subjects are assumed not to make any errors either in encoding or in response. The assumption of error-free encoding is obviously a strong one, although it receives support from data collected by Sternberg and Turner (Note 2).

During the combination stage, errors in information processing are assumed to be due to limitations in subjects' capacities to combine all possible pairs of set relations. In a simply represented pair of premises, such as "No B are C. No A are B," only one possible set relation describes each verbal relation, so that combination of all possible set relations can be easily done. In a complexly represented pair of premises, however, such as "Some B are C. Some A are B," each verbal relation can be represented by four set relations, so that sixteen possible combinations could result. Limitations in mental capacities of subjects solving syllogisms are assumed to result in no more than four set relations ever being combined. The respective probabilities of combining exactly one, two, three, or four pairs of set relations are represented by components p_1, p_2, p_3, and p_4. Since subjects are assumed never to combine

more than four pairs of relations, these probabilities must sum to one.

During the comparison stage, the subject must decide upon an appropriate label for his or her combined set relation. A problem can arise, however, when either of two representations correctly describes the combined set relation. For example, if A and C are equivalent sets, either "All A are C" or "Some A are C" correctly describes the combined set relation. Subjects are assumed to use two heuristics to permit selection of a unique response. In order to describe these heuristics, it is necessary first to introduce some new terminology. Atmosphere of a syllogism is determined by two rules: It is particular (leading to the choice of a particular conclusion) if at least one premise is particular (e.g., Some A are B), and it is negative (leading to the choice of a negative conclusion) if at least one premise is negative (e.g., No B are C). If there is both a particular and a negative in either or both of the premises, the subject selects a particular negative conclusion. Strength of a verbal label is determined by a single rule: A given label is stronger than another if it has fewer possible set relations in its representation. Thus, for example, "All A are C" is stronger than "Some A are C" because the universal statement can be represented by only two set relations (equivalence and set-superset), whereas the particular statement can be represented by as many as four set relations (equivalence, set-superset, set-subset, set overlap). It is now possible to describe the two heuristics permitting selection of a unique response. The first heuristic is that the subject chooses the label that matches the atmosphere of the premises if one label matches the atmosphere of the premises but is weaker than the alternative label. The second heuristic, used if one label both matches the atmosphere of the premises and is the stronger of two labels, is that the subject chooses a label that both matches the atmosphere of the premises and is the stronger of the

two labels. These heuristics are not always used. Thus, the respective probabilities of using them are β_1 and β_2, and the respective probabilities of not using them are $1 - \beta_1$ and $1 - \beta_2$. Finally, there is a maladaptive heuristic subjects adopt with probability c, namely, the labeling of a combined representation as indeterminate if the representation generated during the combination stage includes different initial elements. By initial element is meant the relation between A and B in the relation A r B. (A second element can also be referred to, that between B and A in the relation A r B. The two elements will express the same relation only if the representation is symmetrical, that is, if A is related to B in the same way that B is related to A.)

Experimental data. The transitive-chain theory was tested in a series of experiments using both categorical and conditional syllogisms with two quantified premises (Guyote & Sternberg, Note 1; Sternberg & Turner, Note 2). Some results of these tests are summarized in Table 3. The presentation and

---

Insert Table 3 about here

---

discussion of results will be divided into three parts: fits of mathematical models to data, parameter estimates, and relations of estimated parameters to measured abilities.

The models for predicting response time and response choice were quantified, and the resulting mathematical models were fit to the data using least squares. Proportions of variance in the data accounted for by the mathematical models are shown in the first panel. Note that different kinds of premise content were used in the testing of the response choice model. Abstract content (used in two experiments) denotes premises of the form "All A are B," "No A are B," etc. An example of a factual premise is "All robins are birds."

An example of a counterfactual premise is "All birds are robins." An anomalous premise is one like "All birds are chairs." In "most-few" items, the quantifier some was replaced by either most or few, as in "Few birds are robins." It can be seen from the data that in each case, the transitive-chain models provided a good fit to the response-choice and response-time data. Moreover, the fit of the response-choice model was in every case superior to that of four alternative models also tested on these data. (No alternative response-time models have been previously proposed.) Median values of R^2 for these alternative models were .76 for a "complete combination model" (Erickson, 1974), .59 for a "random combination model" (Erickson, 1974), .57 for an atmosphere model (Woodworth & Sells, 1935), and .78 for a conversion model (Chapman & Chapman, 1959). Although the transitive-chain model was superior to these alternative models, neither the response-choice model nor the response-time model was "true": The variance unexplained by each model was statistically significant in every case. Thus, we still have more to learn about how subjects solve syllogisms with two quantified premises.

The next panel shows parameter estimates for the response-choice and response-time models. In the response-choice model, p_2, p_3, and p_4 were combined because they were estimated from independent variables that were highly correlated. There are four major patterns of results that are worthy of note. First, subjects tend to combine more set relations with factual premises than with other types of premises. This result seems sensible, in that processing of premise information that was previously stored in long-term memory (such as "all robins are birds") seems to require less mental capacity than processing of premise information that must be newly encoded (such as "all robins are chairs"). Second, values of β_1 were always greater than .5, suggesting that when subjects must choose between two labels, one of which matches the

atmosphere of the premises and the other of which is the stronger label, subjects prefer the label that matches the atmosphere of the premises. Third, values of β_2 were close to 1, suggesting that when a given label both matches the atmosphere of the premises and is the stronger label, it is almost always chosen. Fourth, values of c were lower than .5 except in the case of conditional syllogisms: With categorical syllogisms, it appears, subjects are less likely incorrectly to select the "Indeterminate" label than they are with conditional syllogisms.

Parameter estimates for the information-processing model of response time also make sense. Three patterns of results are worthy of note. First, as predicted, encoding of more complex set relations took more time than encoding of simpler set relations. Second, combination of pairs of more complex set relations took more time than combination of pairs of simpler relations. Third, most of the difficulty in syllogisms with two quantified premises is constant across variations in item type, as is shown by the high parameter estimate for the response component. This parameter is estimated as the constant in the regression equation, and thus includes any processes that are common across various item types.

Finally, we turn to relations between estimated parameters and mental ability test scores. We were interested in testing two hypotheses. The first was that interesting individual differences in syllogistic reasoning (i.e., those related to mental test performance) would be found in performance during combination but not comparison: The response-choice components of the combination stage reflect ability to store and manipulate large quantities of information in working memory, whereas the components of the comparison stage merely reflect response biases. The second hypothesis was that any interesting individual differences obtained would be related to spatial-abstract ability

but not verbal ability. Spatial-abstract ability refers to the subject's facility in visualizing and manipulating abstract symbols in his or her head, which is what seems to be required during the combination stage of syllogistic reasoning. Verbal ability refers to the subject's facility in comprehending and reasoning with relations between words. But such relations seem unimportant beyond the initial encoding of the verbal relations, which, according to our model, is always done completely and correctly and is thus not a source of individual differences. The results of our data analyses bore out both hypotheses. Subjects were split into four groups, depending upon whether they were above or below the median on orthogonal factors measuring verbal and spatial-abstract ability. Parameters were estimated for each of the four groups, and mean differences were then computed between groups. The significance of the differences was then tested using a jackknife procedure (see Mosteller & Tukey, 1977). Since $p_2+p_3+p_4 = 1-p_1$, only the p_1 parameter from the combination stage was compared across groups. As can be seen in Table 3, the p_1 parameter, but none of the comparison parameters, showed a significant difference in value across groups of differing spatial-abstract (but not verbal) ability.

Models for One Quantified Premise

We turn now to the application of the transitive-chain theory of syllogistic reasoning to syllogisms with one quantified premise (node IA2 in Figure 1). We shall consider these problems in less detail than the preceding ones, since the theory requires only minor augmentation to deal with them.

Nature of tasks. Syllogisms with one quantified premise, like those with two quantified premises, can be expressed in either categorical or conditional form. The problems differ from the preceding ones in two major respects. First, the second (minor) premise describes the relation between an individual and a

set rather than between two sets. For example, instead of a premise such as "All A are B," one might find the premise "X is an A." Second, the subject's task is to indicate whether a single presented conclusion is logically valid, rather than to choose one of a set of alternative conclusions. The single conclusion, like the second premise, expresses a relation between an individual and a set, for example, "X is a B." Examples of categorical and conditional syllogisms with one quantified premise are the following:

All A are B.	If A then B.
X is an A.	A.
_____	_____
X is a B.	B.

Subjects find syllogisms with one quantified premise considerably easier than they find syllogisms with two quantified premises. Mean response time was 13.45 seconds with a standard deviation of .70 seconds.

Information-processing model. Solution of these syllogisms is theorized to occur in three stages: encoding, combination, and response. There is no comparison stage, since no selection of alternative labels is involved. Two basic strategies are asserted to be involved in solving the problems. The first is the method of direct proof whereby the subject attempts to combine the asserted information in the second (minor) premise with the conditional information in the first (major) premise. It is possible for the method of direct proof to fail to yield a valid conclusion, however, and yet for the syllogism to be valid. The second strategy, a method of indirect proof, is sometimes used when the first strategy fails. The subject negates the conclusion, and attempts to combine the negated assertion of the conclusion with the conditional information in the first premise. If the result contradicts the information in the second premise, the syllogism is valid. (These strate-

gies are described in greater detail in Guyote & Sternberg, Note 1.)

The components of information processing theorized to be used in solving the syllogisms are listed in Table 4. Although there are nine components,

Insert Table 4 about here

not all of these are ever relevant to the solution of a single problem. The components follow a $2^3 + 1$ scheme. The added component is that for response production, and occurs in the last stage of processing. The encoding-combination components are distinguished by whether they apply to (a) the first element (A to B relation in A r B) or second element (B to A relation in A r B) of the set relation(s) implied by the first premise, (b) problems with no negations in the first premise (e.g., All A are B) or one or more negations in the first premise (e.g., No A are B), and (c) use of direct or indirect proof. In the examples presented above, the relevant components would be p_{1p}, s_{1p}, and RESP, since there are no negations in the first premise and indirect proof is not needed to recognize the presented conclusion as valid.

Response-choice model. Components of the response-choice model are listed in Table 5. All components occur in the combination stage, since en-

Insert Table 5 about here

coding and response are assumed to be executed flawlessly. The first two components, whose probabilities of occurrence are represented by p_1 and p_2, correspond to the analogue components from the model for two quantified premises. Note that p_3 and p_4 are omitted from the list. In these simpler problems, in which the first premise is always universally quantified, there are never more than two possible set relations to combine, and thus p_3 and p_4 are irrelevant.

The next three components occur with probabilities t_0, t_1, and t_2. These are probabilities of using the method of indirect proof as a function of the number of negations in the first premise (which may be zero, as in "All A are B" or "If A then B;" one, as in "No A are B" or "If A then not B;" or two, as in "No A are non-B" or "If not A then not B"). Each occurrence of a negation in the first premise increases demands upon processing capacity, and processing capacity consumed by comprehension of negations is then lost to use for indirect proof. Thus, the probability of using indirect proof should decrease with the presence of additional negations in the first premise.

Experimental data. Again, presentation and discussion of data will be divided into three parts: fits of mathematical models to data, parameter estimates, and relations of estimated parameters to mental ability test scores. Results are presented in Table 6.

Insert Table 6 about here

Values of R^2 were very high for both response-choice and response-data, supporting the models. The residual variance was significant in each case, however.

Two predictions were made for the response-choice parameter estimates. First, the relative ease of these syllogisms compared to the previous ones should result in a tendency for subjects to combine more set relations for these than the previous syllogisms. Hence, one would expect the value of p_1 to be lower for syllogisms with one quantified premise than for syllogisms with two quantified premises. Comparison of the values of p_1 in this table with those in Table 3 (for syllogisms having abstract content) supports this prediction. Second, one would expect a monotone decrease in probabilities across t_0, t_1, and t_2, reflecting the decreasing likelihood of a subject's using indirect proof as the number of negations increases. This prediction was only partially supported. Estimated probabilities for t_0 and t_1 are about

the same, with a sharp dropoff for t_2. Apparently, the presence of a single negation does not consume processing capacity to an extent that interferes with utilization of the indirect proof strategy. The presence of a double negation does consume enough capacity, however, to interfere with use of indirect proof.

Consider next the response-time parameters. Three predictions were made regarding their relative values. First, we would expect components based upon the use of the method of indirect proof (subscript of 2) to take longer in execution than the corresponding components based upon the use of the simpler method of direct proof (subscript of 1). This prediction was upheld. Second, we would expect components involving processing of negations (subscript of n) to take longer in execution than the corresponding components involving processing of positive statements (subscript of p). This prediction was also upheld. Finally, we would expect components based upon the first or primary element in the set relation representation (p components) to be executed more rapidly than the corresponding components based upon the second element in the set relation representation (s components). This third prediction was also upheld. Thus, the response-time parameter estimates make good sense in the context of the theoretical model from which they were derived.

Finally, consider the relations of the estimated response-choice parameters to scores on mental ability tests. Subjects were again divided into four crossed groups that were either high or low on either verbal or spatial-abstract ability. Since p_1 ($= 1-p_2$), t_n, t_1, and t_2 are all combination stage parameters, a significant difference was expected between the mean values of high and low spatial-abstract (but not verbal) subjects. Such differences were in fact obtained. These results, then, confirm those of the analyses for syllogisms with two quantified premises.

Union of Models for One and Two Quantified Premises

Consider next node IA in the hierarchy, which represents the point at the next higher level of the hierarchy at which nodes IA1 and IA2 converge. What kind of task would require a union of at least most of the components that are required for the two types of syllogisms (one and two quantified premises) that we have previously considered? The categorical and conditional syllogisms that follow seem to represent this union:

All B are C.	If B then C.
All A are B.	If A then B.
X is an A.	A.
_____	_____
X is a C.	C.

Note that this type of syllogism, which to my knowledge has not been previously investigated, contains two minor premises, one expressing a quantified relation between two sets, the other expressing a nonquantified relation between an individual and a set. Solution of the syllogism seems to require a union of the encoding, combination, and response components of the models for one and two quantified premises. Comparison components are probably not needed, since one may proceed to combine the conclusion of the first two premises with the third premise without an intervening label.

Consider first the components of a plausible information-processing model. The subject must encode the two quantified premises, using the same set of components as is applicable to standard syllogisms with two quantified premises. Next, the subject must combine information from these two premises, using the components applicable to syllogisms with two quantified premises. Then the subject must encode the third premise and combine it with the combined representation from the first two premises, using the components of the model for syllogisms with one quantified premise. Finally, the subject must respond.

Consider next the components of a plausible response-choice model. The probabilities p_1, p_2, p_3, and p_4 are relevant to the combination of the first two premises. Probabilities p_1 and p_2 are also relevant to combination of the outcome of this first set of combinations with the relation expressed by the third premise. It would seem that five parameters--t_0, t_1, t_2, t_3, and t_4--might be needed to represent the probabilities of a subject's using indirect proof as a function of the number of negations in the first two premises.

I have not yet investigated this task in my laboratory, although an analysis of this task is high on the list of priorities for research during the coming year. Successful analysis of the task is expected to show that the models for one and two quantified premises are special cases of a more general model that can encompass both.

Mixture Theory of Transitive Inference

Mixed Model of Linear Syllogistic Reasoning

We turn next to node IB1 in the hierarchy depicted in Figure 1. This node has as its task content the linear syllogism.

Nature of task. A linear syllogism contains two premises and a question. Each of the premises describes a relation between two items, with one of the items overlapping between the two premises. The subject's task is to use this overlap to determine the relation between the two items not occurring in the same premise. Determination of this relation enables the subject to answer the question. An example of a linear syllogism is the following:

C is not as tall as B.
A is not as short as B.

Who is shortest?

Information-processing model. The proposed model of information processing is described in detail in Sternberg (Note 3). The present account is taken from Sternberg, Guyote, and Turner (Note 4).

According to the proposed model, two types of representations are used in the solution of linear syllogisms (and hence the name "mixed model"). First, subjects are hypothesized to decode the premises of the linear syllogism into a linguistically-based, deep-structural proposition of the type originally proposed by Chomsky (1965). A premise such as "John is taller than Mary," for example, would be represented as (John is tall+; Mary is tall) (see Clark, 1969). Next, subjects are hypothesized to recode the deep-structural representation into a spatial array that functions as an internal analogue to a physically realizable array. In such an array, John would be placed above Mary, $\begin{matrix}\text{John}\\ \text{Mary}\end{matrix}$.

According to the mixed model, as many as 10 component processes may be required to solve linear syllogisms of various kinds. These components are summarized in Table 7, and will be illustrated with reference to the example problem cited above.

Insert Table 7 about here

---

1. Premise reading (mandatory). The subject reads each of the two premises, "C is not as tall as B" and "A is not as short as B," comprehending their surface structure.

2. Linguistic decoding of comparative relation (mandatory). The subject decodes the surface-structural form into a deep-structural proposition relating the two terms of the premise. Decoding of a premise with a marked adjective (such as short) is assumed to take longer than decoding of a premise with an unmarked adjective (such as tall). In the example, the first premise is decoded into the form (C is tall+; B is tall); the second premise is decoded into the form (A is short+; B is short). Note that at this point, only the

comparative and not the negative has been processed, so that the deep-structural propositions do not accurately represent the content of the premises.

3. Decoding of negation (optional). If a premise is a negative equative, that is, one with the relation "not as ___ as," it is necessary to reformulate the deep-structural decoding of the premise to take the negation into account. The roles of the terms in the propositions are reversed, so that the first proposition becomes (B is tall+; C is tall) and the second one becomes (B is short+; A is short).

4. Spatial seriation of comparative relation (mandatory). Having decoded the premises into deep-structural propositions, the subject is now able to seriate the terms of each premise spatially. A propositional encoding is assumed to be prerequisite for spatial seriation. The subject may seriate the two terms of each premise in either a preferred (usually top-down) or nonpreferred (usually bottom-up) direction. It is assumed that the subject's choice of direction depends upon whether or not the adjective in the original premise was marked or not. The preferred direction is used for unmarked adjectives, the nonpreferred direction for marked adjectives. In the example, B and C are seriated top-down into one spatial array, $\frac{B}{C}$. B and A are seriated bottom-up into a second spatial array, $\frac{A}{B}$.

5. Pivot search (optional). Once the subject has seriated the terms in each of the two premises into two spatial arrays, the subject must locate the middle (pivot) term that will enable him or her to combine the two arrays into a single array. The pivot is assumed to be immediately available if either (a) it appears in two affirmative premises, or (b) it was the last term to be seriated in a negative equative. (The principles behind this availability are described in Sternberg, Note 3). In the example, the last

term to have been seriated was A (the tallest term). The subject inquires whether A is the pivot. Since it is not, the subject must use additional time locating the pivot, B, which is the only term that appears in both premises.

6. Seriation of the two arrays into a single array (mandatory). Having found the pivot, the subject is prepared to combine the two separate arrays into a single, integrated spatial array. The subject combines the two single arrays according to the order of the original premises. Combination of these arrays is assumed to be less susceptible to error (although not less time-consuming) if the first term to be combined (which is always the first term in the final deep-structural proposition describing the first premise) is the term that is most current in working memory, namely, the pivot (from the immediately preceding operation 5). In the example, the subject starts seriation with the B term as encoded from the bottom half of the array, $\frac{B}{C}$, and ends up in the top half of the array, $\frac{A}{B}$. Thus, the subject links the second pair of terms, A and B, to the first pair, C and B, forming the spatial array, $\begin{matrix} A \\ B. \\ C \end{matrix}$

7. Question reading (mandatory). Next, the subject must read the question that he or she will be required to answer. If the question contains a marked adjective, as does the question in the example, it is assumed to take longer to decode, and the subject is assumed to have to search for the response to the question in the nonpreferred end of the array. A marked adjective in the question, therefore, increases response latency. The example question, "Who is shortest?", contains such an adjective.

8. Response search (optional). After seriation was completed (Operation 6), the "mind's eye" of the subject ended up either in the top or bottom half of the spatial array. If the question has as its answer the term that is in the half of the array in which the subject's mind's eye ended up, then the response

is immediately available. If the answer term is in the other half of the array, however, then the response is not available and must be sought. This search requires additional time. In the example, the subject ended up in the top half of the array, completing seriation with the A and B terms. The question, however, asks who is shortest. The subject must, therefore, search for the response, finding it in the bottom half of the array.

9. Establishment of congruence (optional). The processes described above are sufficient to establish a correct answer, and under some circumstances, a response is immediately forthcoming. If, however, subjects wish to check the accuracy of the response obtained by interrogation of their spatial array, they have available to them their propositional representation by which they can verify their response. If the linguistic encoding of the proposed response is congruent with the linguistic encoding of the corresponding term of the proposition, then the response immediately passes the congruence check. If the two are incongruent, however, congruence of the response term to the propositional term is established, taking additional time. In the example, C, the shortest term, was described as tall (relative to B, who was tall+). The question, however, asks who is shortest. Congruence must therefore be established by formulating the question in terms of who is least tall.

10. Response (mandatory). The final operation is response, whereby the subject communicates his or her choice of an answer. In the example, the subject responds with C.

Error model. Under standard instructions telling subjects to work as quickly as they can without making errors, subjects make very few errors (about 1% of responses), so that there is little basis for modeling choice of responses. Under instructions that emphasize speed at the expense of accuracy, however (Sternberg, Note 5), error rates can be boosted to about

7%. In this task, a mathematical model directly based upon the information-processing model was used to predict errors. Proportion of response errors was postulated to equal the appropriately scaled sum of the difficulties encountered in executing each component. A simple linear model predicts proportion of errors to be the sum across the different information-processing components of the number of times each component is performed (as an independent variable) multiplied by the difficulty of that component (as an estimated parameter). This additive combination rule is based upon the assumption that each subject has a limit on processing capacity (see Osherson, 1974; Sternberg, 1977b). Each execution of an operation uses up capacity. Until the limit is exceeded, performance is flawless except for constant sources of error (such as motor confusion, carelessness, momentary distractions, etc.). Once the limit is exceeded, however, performance is at a chance level. The relation between the error model and the response-time model is somewhat complex, and is described in Sternberg (Note 5).

Experimental data. As in previous analyses, we shall be concerned with fits of the mathematical models to the data, parameter estimates, and relations of parameter estimates to mental ability test scores. The data are presented in Table 8.

Insert Table 8 about here

The top panel of the table shows that the mixed model gives a good account of the response-time data under a variety of experimental conditions. These conditions differed in whether (a) the problem was presented as a whole (Experiments 3, 4, 5) or split into parts (Experiments 1, 2), (b) the question was presented first (Experiment 2) or last (Experiments 1, 3, 4, 5), (c) three different adjective pairs (taller-shorter, better-worse, faster-slower) were

presented within subjects (Experiments 1, 2, 3, 5) or between subjects (Experiment 4), and (d) instructions emphasized both speed and accuracy (Experiments 1, 2, 3, 4) or primarily speed (Experiment 5). The mixed model was also compared to alternative linguistic and spatial models based upon proposals of Clark (1969) (linguistic) and of DeSoto, London, and Handel (1965) and Huttenlocher (1968) (spatial). The differences in R^2 between the mixed model and the second best model (which was the linguistic model in the first four experiments and the spatial model in the fifth) were .213, .148, .155, .240, and .237 in Experiments 1, 2, 3, 4, and 5 respectively, suggesting that the mixed model is superior to its competitors. The residual variance was statistically significant, however, in four of the five experiments, indicating that the mixed model is not identical to the true model of how subjects perform.

None of the experimental designs permitted estimation of all parameters of the mixed model. The table shows those parameters that could be estimated from at least several experiments. Some of these parameters are confounded, as indicated in the table Note. It can be seen that most of a subject's time is spent in operations that are constant across variations in item types (RES+) and in encoding operations (ENC+). It is worth noting that for the most part, the group parameter estimates were reasonable and in close agreement across data sets. Component times were thus relatively stable across experimental conditions, except for the speeded one, where, as expected, they were of generally shorter duration than in the other conditions.

The bottom panel of Table 8 shows correlations of parameter estimates for individual subjects with composite scores on tests of verbal, spatial, and abstract reasoning abilities. Data are combined across the first four experiments. It was predicted that parameters representing durations of verbal components would be significantly correlated with verbal but not spatial or abstract tests; that parameters representing durations of spatial components would be significantly

correlated with spatial and abstract but not verbal composites; and that confounded components would be correlated with each type of test in rough proportion to the contribution of each kind of pure component to the confounded component. Specifically, significant correlations were expected between the verbal composite and encoding, negation, marking, noncongruence, and response; and significant correlations were expected between the spatial and abstract composites and encoding, marking, pivot search, and response search. The obtained pattern of correlations was as predicted with two exceptions. Negation turned out to be a spatial rather than a linguistic component, suggesting that it is accomplished by flipping terms around in a spatial array rather than by reformulating deep-structural propositions. Response search turned out to be significantly correlated with the verbal as well as the spatial and abstract composites, suggesting the possibility of nontrivial individual differences in the reading off of verbal labels (names) from spatial arrays. Given the number of possible outcomes that might have derived, these correlational data seem to provide good support for the assumptions of the mixed model regarding the type of representation upon which each component operates.

Augmented Mixed Model for N-Term Linear Syllogisms

The mixed model of linear syllogistic reasoning (node IB1) is believed to be a special case of a more general mixed model that can be applied to linear syllogisms with N terms. We are currently investigating in my laboratory problems of the following form, where the subject's task is to indicate which of the two terms at the bottom of the problem is taller. In the actual problems, names are substituted for letters:

A is taller than B.

C is taller than D.

C is shorter than B.

B C

We expect the mixed model for standard linear syllogisms to require augmentation to handle problems of this type, although the nature of the augmentation that will be required is not yet clear.

Union of Transitive-Chain and Mixture Theories

What kind of task might represent a union of at least most of the components of the transitive-chain theory of syllogistic reasoning on the one hand and the mixture theory of transitive inference on the other? Such a task would provide the basis for an analysis of the components of information processing required at node I of the hierarchy in Figure 1.

We are currently investigating in my laboratory two variants of a task that seems to represent a union of required components. We call the task a quantified linear syllogism. It can be presented in either of two forms:

All C are not as tall as some B.

Some A are not as short as all B.

(A) All A are taller than all C.

(B) All A are taller than some C.

(C) Some A are taller than all C.

(D) Some A are taller than some C.

(E) Indeterminate

All C are not as tall as some B.

Some A are not as short as all B.

Which are shortest?

(A) All A (D) Some B

(B) Some A (E) All C

(C) All B (F) Some C

(G) Indeterminate

In the actual problems, nonsense syllables were substituted for letters.

There seem to be two basic relationships between quantified linear syllo-

gisms and the types of syllogisms (categorical, conditional, linear) considered above:

1. Quantified linear syllogisms are like categorical and conditional syllogisms in their use of quantifiers in the premises, but like linear syllogisms in their requirement of a transitive inference. In these new problems, the quantification of the second term in each premise is made explicit, whereas in standard syllogisms this quantification is left implicit (particular for affirmative premises, universal for negative ones). The quantification is made explicit to avoid ambiguity.

2. The first variant of the quantified linear syllogisms task is like categorical and conditional syllogisms in requiring the subject to select the conclusion that follows from the premises, or else to indicate an indeterminacy. In this variant of the problem, the subject is told which terms to relate (A and C), but must discover how to relate them. The second variant is like linear syllogisms in requiring the subject to answer a question--which term is shortest (or tallest)? In this variant of the problem, the subject is told what relation is of interest (shortest or tallest), but must discover which term satisfies this relation.

At first glance, quantified linear syllogisms appear to constitute an unusual task. To my knowledge, they have not been previously investigated in an experimental setting. However, I believe they have more ecological validity than any of the standard syllogisms. Consider, for example, statements like "College graduates are brighter than college dropouts" or "Brain surgeons make more money than college professors." These statements seem to contain implicit universal quantifiers before both their subjects and predicates, and as fully universally quantified statements, are false. Yet, it is certainly the case, for example, that "Some college graduates are brighter

than some college dropouts" and that "All college graduates are brighter than some college dropouts." It may even be the case that "Some college graduates are brighter than all college dropouts." When dealing with natural classes of objects or events, it seems seldom to be the case that all members of one class possess more of some attribute than all members of another class. We therefore need to quantify our relational statements explicitly, consoling ourselves, perhaps, that at least some college professors make more money than some brain surgeons.

COMPONENTIAL THEORY OF INDUCTION

IMAJER Theory

Models for Integral Stimuli

We will begin our consideration of the theory of induction with node IIA1, the IMAJER theory for integral (unitary) stimuli. IMAJER is an acronym for the six components presented in Table 9 (inference, mapping, application, justification, encoding, response). These six processes are theorized to be involved in a wide

---

Insert Table 9 about here

---

variety of inductive reasoning tasks of the sort found on intelligence tests, including analogy, classification, series completion, topological relations, and metaphor. The nature and generality of the components seem best described by illustrating their relevance to the solution of these tasks, and so the use of each component in each task will be described.

Analogy. Consider the component processes a subject might use in solving an analogy such as

Truman : Eisenhower :: Louis XIII : (a) Louis XIV, (b) Robespierre.

The subject would seem to have to encode the terms of the analogy, translating each stimulus into an internal representation upon which further mental operations can be performed. The internal representation is stored in working memory, and is assumed to consist of an attribute-value list. The subject

must also infer the relation between Truman and Eisenhower, recognizing, perhaps, that Truman was the predecessor of Eisenhower as president. Then the subject must map the relation between Truman and Louis XIII, realizing that both were heads of state. Next, the subject must apply the relation that was previously inferred from Louis XIII to each of the answer options, deciding which option bears the same relation to Louis XIII as Eisenhower does to Truman. Optionally, the subject may justify one of the options as preferred but nonideal. If, for example, a subject believes that Louis XIV followed Louis XIII, but was not his immediate successor, then the subject may view Louis XIV as the preferred option, but not the ideal one. Finally, the subject must respond, communicating his or her answer.

Classification. We shall consider two kinds of classification, simple and complex. Operationally, these two kinds of problems differ in whether or not they require a mapping component.

Consider the simple classification

Eisenhower, Louis XIII, Truman (a) Louis XIV, (b) Robespierre.

The subject's task is to indicate which of the two terms at the right belongs with the three terms at the left.

In order to solve the problem, the subject must encode the terms of the classification. He or she must also infer what is common to Eisenhower, Louis XIII, and Truman, perhaps that they were all heads of state. The subject must then apply this relation to the two answer options, deciding which belongs with the three items in the stem. Optionally, the subject may justify one of the answer options as preferred but nonideal. Justification would be needed, for example, if Louis XIV were viewed as not quite mortal and hence in a different league from the three ordinary, mortal heads of state. Finally, the subject must respond, indicating his or her preferred answer.

Next consider the complex classification

(a) Eisenhower, Truman (b) Louis XIII, Louis XIV

Robespierre

Here the subject's task is to decide whether Robespierre belongs with the terms in group (a) or with the terms in group (b).

Again, the subject must encode the terms of the classification. He or she must also infer what attributes are common to Eisenhower and Truman, and what attributes are common to Louis XIII and Louis XIV. Next, the subject may map the differences between groups (a) and (b), arriving at a list of distinguishing attributes. In this example, the list might include two items, such as that group (a) consists of Americans and group (b) of Frenchmen, and that group (a) consists of elected heads of state and group (b) of heads of state appointed by virtue of inheritance. Then the subject must apply to Robespierre the rules inferred for groups (a) and (b). He or she need only consider, however, the subset of mapped attributes that distinguishes group membership. The subject may optionally justify membership in one group as preferred but nonideal. For example, Robespierre seems better to belong in group (b) because he was French; but since he was not a head of state, he is not a perfect fit for that group. Finally, the subject responds.

Series completion. Series completion items, like classification items, may be either simple or complex, depending upon whether or not the subject needs to map a relation from one domain to another.

Consider an example of a simple series completion problem:

Truman, Eisenhower, Kennedy, (a) Johnson, (b) Roosevelt.

The subject's task is to indicate which of the two answer options should come next in the series.

As in the previous item types, the subject must encode the terms of the series completion problem. The subject must also make two inferences. The first is an unrestricted inference of the relation between Truman and Eisenhower. The second is a restricted inference of the relation between Eisenhower and Kennedy. I call the inference restricted because it need consist only of a subset of those attributes that were inferred between the first two terms. If none of these successfully relate the second to the third term, then the first inference must have been inadequate, since it did not permit continuation of the series. In this event, this first inference will have to be revised. Next, the subject must apply the rule he or she has inferred to both Johnson and Roosevelt. Optionally, the subject may justify one or the other option as preferred but nonideal. If, for example, the subject did not realize that Johnson immediately succeeded Kennedy, Johnson might be viewed as preferred but nonideal. Finally, the subject must respond.

A complex series completion problem takes the following form:

Truman, Eisenhower, Kennedy;

Louis XIII, (a) Louis XIV, (b) Robespierre.

Here, the subject's task is to complete the series from the first term in the second line, rather than from the last term in the first line.

The subject must encode the terms of the series completion problem. He or she must also infer the unrestricted relation between Truman and Eisenhower, and then the restricted relation between Eisenhower and Kennedy. Next, the subject must map the relation between Kennedy and Louis XIII. Then the subject must apply the rule he or she has inferred from Louis XIII to each answer option, selecting the preferred one. Optionally, the subject may need to justify one or the other option as preferred. Finally, the subject must respond.

Topological relations. Figure 2 shows an example of a topological relations problem. In this type of problem, the subject must ascertain

---

Insert Figure 2 about here

---

the relation of the dot to the other objects in a target figure. The subject must then choose the answer option that allows placement of the dot in a position topologically equivalent to its placement in the target figure.

To solve such a problem, the subject must encode the terms of the problem, noting, for example, that the target figure consists of a square, a line, a triangle, and a dot. Next, the subject must infer the relation between the dot and the other elements in the figure. For example, the dot in the target figure is below the line, outside the triangle, and inside the square. Next, the subject must map the relation between the target figure and each of the answer options, realizing, for example, the correspondences between the squares, lines, and triangles in the target figure and the two options. Then the subject must apply the inferred relation from the target figure to the answer options, inquiring in which figure it is possible to place a dot that meets the same topological constraints as the dot in the target figure. Optionally, the subject may justify one answer as preferred if the dot meets more of the required constraints in one figure than in the other, but meets all of the constraints in neither figure. Finally, the subject responds.

Metaphor. In a metaphor, a subject must comprehend figurative statements such as the following:

The moon in the sky is a ghostly galleon upon the sea.

The moon in the sky is a ghostly galleon.

The moon is a ghostly galleon.

Consider the first metaphor. What components are required for comprehension of the metaphor? First, the subject must encode the terms of the metaphor. Next, the subject needs to infer the relation between moon and sky. Then the subject must map the relation from moon to ghostly galleon. And then the subject must apply the inferred relation from ghostly galleon to sea. Optionally, the subject may justify sea as acceptable but nonideal if its role in the metaphor does not quite match that of sky. Finally, the subject must make some response. We have required either of two kinds of response in the task as studied in my laboratory. The subject is asked to provide a rating either of (a) comprehensibility or (b) goodness of the metaphor.

The second and third metaphors differ from the first one in the absence of certain terms, which are left implicit rather than being made explicit. In the second metaphor, the fourth term is missing, and in the third metaphor, both the second and the fourth terms are missing. We hypothesize that comprehensibility will decrease with missing terms, because subjects are required to insert these terms themselves and may have difficulty in doing so, but that goodness will increase, because the metaphor seems to retain more aesthetic appeal when at least part of it is left implicit for the subject to insert.

Information-processing models. The descriptions of information processing provided above are at a global level, and do not purport to specify fully the steps subjects use in combining (a) different components and (b) multiple executions of the same components. The information-processing models for various types of analogies are fully specified in Sternberg (1977b), and are specified in less detail in Sternberg (1977a). Models for each of the other tasks (except metaphor) are fully specified in Sternberg (Note 6).

Each task has been or now is being analyzed by each of four alternative models of information processing, all of which fall under the general IMAJER

theory. In Model I, inference, mapping, and application are all exhaustive. In other words, the subject always infers, maps, and applies all attributes that he or she has encoded into working memory. In Model II, inference and mapping are exhaustive, but application is self-terminating. In other words, the subject always infers and maps all encoded attributes, but only applies as many attributes as are needed to yield a unique response. In Model III, inference is exhaustive but mapping and application are self-terminating. Here, the subject infers all encoded attributes, but maps and applies only the minimum subset needed to choose a unique response. And in Model IV, inference, mapping, and application are all self-terminating. The subject infers, maps, and applies only those attributes needed to select a unique response. The four models apply to all of the tasks, although with slight modifications due to specific characteristics of each task.

Error models. Error models were derived for these tasks in the same way that they were derived for the linear syllogisms task. Each source of difficulty in the task is assumed to contribute linearly toward the total difficulty of a given item. Up to a certain level of difficulty, subjects are assumed to perform correctly except for constant sources of error. Beyond this level of difficulty, subjects are assumed to perform at a chance level. Thus, a linear model predicts error rates from the components of the four information-processing models.

Experimental data. Although data have been collected from 11 experiments analyzing all of these types of tasks, the only data that have yet been analyzed are those for previous experiments on analogical reasoning. Because of the enormous size of the data base, it may be a number of months before the remainder of the data are fully analyzed. A preliminary report for the analogy data thus seems in order. These data are presented in Table 10.

Insert Table 10 about here

The top panel of the table shows fits of the preferred model of analogical reasoning (III) to response-time and error data. The values of R^2 show that the model provided a good fit to response-time data for each of the three types of content on which the models were tested. In view of the small amount of error data (1½% for People Pieces, 5% for verbal and geometric items), the fits to the error data are also respectable, except for the verbal analogies. The IMAJER theory was compared to two alternative theories (see Sternberg, 1977a), one of which lacked the mapping component, the other of which combined the inference and application components into a single component. The data did not support the theory lacking the mapping component, but failed to distinguish between the other two theories. Regardless of the theory or type of content, however, there was always systematic variance left unexplained, meaning that none of the theories can be considered true.

Parameter estimates for the various components of the theory are shown in the second panel. Four aspects of these data are worthy of note. First, encoding was the most time-consuming process for all three types of content, and took successively longer for People Piece, verbal, and geometric analogies. Second, the response component was of about equal duration for each type of content, as would be expected, since according to the theory the amount of time taken in response should be constant across item contents. Third, inference, mapping, and application--the three attribute-comparison components--were relatively rapid for People Piece and verbal analogies, but relatively slow for geometric analogies, suggesting that in these types of analogies, discovering the relevant attributes was considerably more time-consuming than in the other two types of analogies. Fourth, the components that contributed

significantly to the error model, except for encoding in the geometric analogy experiment, were self-terminating ones. Apparently, when subjects make errors, the errors are due almost exclusively to errors in self-terminating rather than exhaustive processes. By terminating processing of these components early, subjects probably achieve greater rapidity of processing at the expense of increased errors in information processing.

Finally, we turn to the relations of the estimated parameters of the model to scores on reasoning tests. Correlations between individual subjects' parameter estimates and their scores on a reasoning test composite are shown in the bottom panel of the table. Three patterns of results are of particular interest. All three patterns are rather unusual, and are discussed in greater detail in Sternberg (1977b). First, inference, mapping, and application appear to correlate with reasoning only when discovery of stimulus attributes is a nontrivial task. In the People Piece analogies, the same, obvious attributes were used on every trial, and in these analogies, no significant correlations were obtained. Second, the encoding parameter is positively correlated with scores on reasoning tests, meaning that longer encoding times are associated with higher reasoning scores. These correlations are even higher when the encoding parameter is standardized within subjects. The positive correlations suggest that better reasoners may follow a strategy whereby they encode the terms of the analogy more carefully and completely than do poorer reasoners, thereby facilitating subsequent component processing on these encodings. Finally, the response (constant) component shows a high correlation with reasoning. This result, which has been replicated by others (e.g., Hunt, Lunneborg, & Lewis, 1975; Lunneborg, 1977; Egan, Note 7), indicates that some process that is constant across the various item types is highly associated with reasoning ability. This constant process may be an

ability to form a strategy, or to execute a strategy, or to perform the numerous bookkeeping operations that are needed to keep track of one's place in the solution of a problem. The result shows the need to decompose the response component into smaller constituents, one or more of which may hold the key to our understanding of the relation between the response component and reasoning.

Models for Separable Stimuli

We consider next node IIA2 in the hierarchy of Figure 1. In a developmental study (Sternberg & Rifkin, Note 8), a colleague and I tested the models of analogical reasoning on stimuli with separable (analyzable) attributes. The stimuli were schematic pictures like the People Pieces, except that any one or more attributes could be deleted without affecting the integrity of the stimulus as a whole. We found three major differences in information processing between these stimuli and the previously investigated integral ones. First, subjects either did not use the mapping component or mapped at a rate that was constant across variations in item type. (These two explanations of the statistically trivial mapping parameter could not be distinguished.) Second, encoding of stimulus attributes was self-terminating rather than exhaustive: Subjects encoded only those attributes of each analogy term that were essential to solution of the analogy. Third, subjects used Model IVM (the analogue for separable stimuli of Model IV for integral stimuli) rather than Model IIIM (the analogue for separable stimuli of Model III for integral stimuli): Inference as well as application was self-terminating. Model fits for Model IVM were comparable to those for Model III when applied to the People Piece analogies. The value of R^2 for adults, for example, was .94 for respone-time data and .65 for error data.

Theory of Causal Inference

We have started investigating in my laboratory the processes people use when making causal inferences (node IIB in the hierarchy of Figure 1). As of the present, we have formulated a model of response choice but not an information-processing model.

Nature of Task

In the causal inference task, subjects receive problems like the following, couched either in abstract content (single letters as events) or sentential content (as below):

1. In City 1, it was observed that
   (a) a sewage line had broken,
   (b) the incidence of stray dogs had increased,
   (c) mosquito control had been abandoned for lack of funds.

   An epidemic of Wilson-Barry Syndrome was reported.

2. In City 2, it was observed that
   (a) the incidence of stray dogs had increased,
   (b) all sewage lines were intact,
   (c) mosquito control had been abandoned for lack of funds.

   An epidemic of Wilson-Barry Syndrome was reported.

3. In City 3, it was observed that
   (a) a radiation leak had occurred in a nuclear reactor,
   (b) the incidence of stray dogs was normal (no increase),
   (c) a sewage line had broken.

   No epidemic of Wilson-Barry Syndrome was reported.

4. In City 4, it was observed that
   (a) mosquito control had been abandoned for lack of funds,

(b) all sewage lines were intact,

(c) incidence of measles was higher than normal.

No epidemic of Wilson-Barry Syndrome was reported.

How likely is it that a broken sewage line, in isolation, leads to an epidemic of Wilson-Barry Syndrome?

Response-Choice Model

Seven components enter into the model of response choice. These components are summarized in Table 11. The first of these components is the

---

Insert Table 11 about here

---

weight assigned to positive affirming instances, as in item 1 (a broken sewage line has been observed and an epidemic has been reported). The second component is the weight assigned to negative affirming instances, as in item 4 (all sewage lines were intact and no epidemic was reported). The third component is the weight assigned to positive infirming instances, as in item 3 (a broken sewage line has been observed but no epidemic was reported). The fourth component is the weight assigned to negative infirming instances, as in item 2 (all sewage lines were intact but an epidemic was reported). The fifth component is the weight assigned to positive affirming evidence for the two strongest distractors, where these distractors are designated to be those for which there is the most affirming evidence and the least infirming evidence (in this example, an increase in the incidence of stray dogs and the abandonment of mosquito control). The sixth component is the weight assigned to positive infirming evidence for the two strongest distractors. The last component is a base likelihood that is assigned regardless of the information contained in any particular problem. These components were combined into a linear model

to predict subjects' estimates of likelihoods (which were provided by subjects on a 0 to 1 scale accurate to two decimal points).

Experimental Data

To date, we have carried out preliminary data analyses on results from a single experiment using abstract content, and are presently engaged in two more experiments, one with abstract content and one with sentential content. The preliminary results from the first experiment are presented in Table 12.

Insert Table 12 about here

The data show that the relatively simple linear model we have tested does a very good job of accounting for variation across items in assigned likelihood ratings. This model accounted for 86% of the variance in the data.

The parameter estimates for the components of the model show that constituent pieces of information were used in the ways one would have expected them to be used. For the target hypothesis, affirming evidence was assigned a positive weight and infirming evidence was assigned a negative weight. In other words, affirming evidence increased the likelihood that the target hypothesis was perceived as correct, whereas infirming evidence decreased the likelihood that this hypothesis was perceived as correct. For the alternative hypotheses, the signs of the weights were reversed. Affirming evidence for an alternative hypothesis decreased the likelihood that the target hypothesis was perceived as correct, whereas infirming evidence for an alternative increased the likelihood that the target hypothesis was perceived as correct. In general, evidence from alternative hypotheses was assigned less weight than evidence from the target hypothesis, as might well be expected. Note the high value (.38) of the base likelihood. This value can be interpreted as indicating that in the absence of any evidence at all, subjects would assign a likelihood

of .38 to the target hypothesis as leading in isolation to the consequent of interest.

The bottom panel shows relations of parameters estimated for individual subjects to a composite reasoning score. None of the correlations were significant, although all were in the expected directions, indicating small but nonsignificant tendencies for subjects higher in reasoning to utilize each component more fully than did subjects lower in reasoning ability.

Union of IMAJER Theory and Theory of Causal Inference

Is there a task belonging at node II of the hierarchy in Figure 1 that somehow requires a union of the components of the IMAJER theory with those of the theory of causal inference? There does appear to be such a task, which we are currently investigating in my laboratory. The stem of the problem (items 1-4) is the same as that of the causal inference problem described earlier. Instead of the question that follows item 4 in that problem, however, the subject finds information and a question of the following kind:

5. In City 5, it was observed that

   (a) mosquito control was operating normally,

   (b) a sewage line had broken,

   (c) the incidence of stray dogs had increased.

   How likely is it that an epidemic of Wilson-Barry Syndrome was reported in City 5?

Note that in this problem, as in the preceding one, the dependent variable is a likelihood rating. However, here the subject must rate the likelihood that an epidemic of Wilson-Barry Syndrome was reported in City 5, rather than the likelihood that a certain antecedent in isolation led to this consequent.

This problem, which I call a causal complex classification, bears an

interesting relation to the complex classification problem described earlier. Recall that in this type of problem, the subject is presented with two items in each of two categories, and is required to indicate in which of these two categories a target item belongs. In the example that was presented, Robespierre was better classified with Louis XIII and Louis XIV than with Truman and Eisenhower. The present problem uses this format in the context of a causal inference problem. The first category consists of two sets of antecedent events occurring in two different cities in which an occurrence of the epidemic was observed. The second category consists of two different sets of antecedent events occurring in two other cities in which an occurrence of the epidemic was not observed. The target consists of a fifth set of antecedent events occurring in a fifth city. The subject must decide how likely it is that an epidemic occurred in this city. Likelihood ratings of greater than .5 indicate a greater affinity to the category in which the epidemic occurred; ratings of less than .5 indicate a greater affinity to the category in which the epidemic did not occur.

To solve problems of this kind, the subject must first encode the terms of the problem. He or she must then infer what antecedents the first two cities have in common that might have led to the epidemic. Then the subject must infer what antecedents the second two cities have in common that are uniform across the two nonoccurrences of the epidemic. Next, the subject may map the differences, selecting out those antecedents that distinguish the first category from the second. Then the subject must apply the inferred rule as mapped between the categories to determine in which category the fifth city belongs. The antecedent events for the fifth city will generally be ambiguous in distinguishing its category membership, and so justification will be used to decide how likely it is that this city belongs in the category of the cities

in which the epidemic occurred. Finally, the subject must respond. In inferring, mapping, and applying causal relations, the subject must weight positive and negative affirming and infirming evidence, thereby using the components of the theory of causal inference as well as those of the IMAJER theory.

UNIFIED COMPONENTIAL THEORY OF HUMAN REASONING

Finally, we come to the top node in the hierarchy of Figure 1. A task at this node would require a union of the components of both deduction and induction. What form might such a task take? We plan to investigate the task depicted in Figure 3 as an example of a task requiring such a union.

Insert Figure 3 about here

In this task, called an induced syllogism, the subject must first encode the set-membership diagrams in the top line showing memberships of individual b's and c's either in unique partitions of sets B and C or in an overlapping partition. Next, the subject must infer a label that describes the relation between set B and set C for all three diagrams. The only label that describes all three relations is that "Some B are C." Next, the subject must encode the set-membership diagrams in the second line, and again infer a label that describes the relation between sets (here A and B) for all three diagrams. Since subjects are told for this task that "some" is to be interpreted as meaning "some but not all," the only label that describes all three relations is that "All A are B." The subject has now inferred two labels, which he or she is to use as the major and minor premises of a syllogism. The subject solves the syllogism, and then must find an answer option with three set diagrams all of which match the label he or she has chosen. In other words, the subject must decide which three diagrams are applications of the chosen label. Finally, the subject must respond with one of the four sets of diagrams, or else respond

that the solution to the problem is indeterminate.

The induced syllogism sandwiches the components of the transitive-chain theory of syllogistic reasoning inside the components of the IMAJER theory of simple classificational reasoning. To see how the "bread" of the sandwich is analogous to a simple classification problem, consider the following: Suppose we (a) ignore the first line at the top of the problem, and (b) convert every b to a c in the second line of the problem. We now ask subjects to select the group of three diagrams from the answer options that represents in each case the same set relation as the group of diagrams from the stem of the item. In this problem, the subject must encode the terms of the problem, infer what label is common to the three set relations in the stem, and apply this relation to determine which set of three set relations in the answer options belongs with the three set relations in the stem. The subject must then respond. We now have a simple classification problem. The induced syllogism differs from this problem by the addition of the first line (premise), and of the intervening syllogism task.

CONCLUSIONS

We have now surveyed at least one task, a theory, and where available, data fitting into each node of the hierarchy presented in Figure 1. The theoretical and empirical enterprise is by no means complete. I expect the structure of the hierarchy to become increasingly more ramified, and the theories at each node of the hierarchy to become increasingly more nearly correct, with the passage of time. Data collection will also need to continue for at least the next several years. But certain conclusions about the theory and its structure can be drawn now, and are not likely to change:

1. Human reasoning can be characterized in terms of a unified theory that comprises hierarchically nested subtheories accounting for performance

on successively more narrow tasks.

2. At the heart of the global theory and each of the subtheories is a relatively small set of components that characterizes the elementary information processes of human reasoning. The components enter into information processing at multiple levels of the hierarchical task structure.

3. The data collected to date are generally consistent with the hierarchical structure proposed in this article. Although none of the accounts of performance are "true" in the sense of accounting for all of the reliable variance in the data, there are no alternative subtheories that are superior to any of the present ones, and there is no alternative account at all that attempts to explain the range of reasoning data explained by the unified componential theory.

The componential approach to understanding human reasoning and intelligence (Sternberg, 1977b, 1978) provides a means by which relations among tasks and among subjects can be understood within a unified framework. It therefore seems to give us a vehicle by which we may understand underlying unities in cognition that escape notice when subjected to fragmentary methodological or theoretical analyses.

Reference Notes

1. Guyote, M. J., & Sternberg, R. J. A transitive-chain theory of syllogistic reasoning (NR 150-412 Technical Report No. 5). New Haven, Conn.: Department of Psychology, Yale University, March, 1978.

2. Sternberg, R. J., & Turner, M. E. Components of syllogistic reasoning (NR 150-412 Technical Report No. 6). New Haven, Conn.: Department of Psychology, Yale University, March, 1978.

3. Sternberg, R. J. Representation and process in transitive inference. Manuscript submitted for publication, 1977.

4. Sternberg, R. J., Guyote, M. J., & Turner, M. E. Deductive reasoning (NR 150-412 Technical Report No. 3). New Haven, Conn.: Department of Psychology, Yale University, January, 1978.

5. Sternberg, R. J. A proposed resolution of curious conflicts in the literature on linear syllogistic reasoning. Manuscript submitted for publication, 1977.

6. Sternberg, R. J. The development of human reasoning. Grant proposal submitted to the National Science Foundation, 1977. The relevant theoretical section will be supplied to readers upon request.

7. Egan, D. E. Characterizing spatial ability: Different mental processes reflected in accuracy and latency scores. Unpublished manuscript, 1977.

8. Sternberg, R. J., & Rifkin, B. The development of analogical reasoning processes. Manuscript submitted for publication, 1977.

References

Chapman, L. J., & Chapman, J. P. Atmosphere effect re-examined. Journal of Experimental Psychology, 1959, 58, 220-226.

Chomsky, N. Aspects of the theory of syntax. Cambridge, Mass.: MIT Press, 1965.

Clark, H. H. Linguistic processes in deductive reasoning. Psychological Review, 1969, 76, 387-404.

DeSoto, C. B., London, M., & Handel, S. Social reasoning and spatial paralogic. Journal of Personality and Social Psychology, 1965, 2, 513-521.

Erickson, J. R. A set analysis theory of behavior in formal syllogistic reasoning tasks. In R. Solso (Ed.), Loyola symposium on cognition (Vol. 2). Hillsdale, N.J.: Erlbaum, 1974.

Hunt, E. B., Lunneborg, C., & Lewis, J. What does it mean to be high verbal? Cognitive Psychology, 1975, 7, 194-227.

Huttenlocher, J. Constructing spatial images: A strategy in reasoning. Psychological Review, 1968, 75, 550-560.

Lunneborg, C. Choice reaction time: What role in ability measurement? Applied Psychological Measurement, 1977, 1, 309-330.

Mosteller, F., & Tukey, J. Data analysis, including statistics. In E. Aronson & G. Lindzey (Eds.), The handbook of social psychology. Reading, Mass.: Addison-Wesley, 1969.

Osherson, D. N. Logical abilities in children (Vol. 2): Logical inference: Underlying operations. Hillsdale, N.J.: Erlbaum, 1974.

Sternberg, R. J. Component processes in analogical reasoning. Psychological Review, 1977, 84, 353-378. (a)

Sternberg, R. J. Intelligence, information processing, and analogical reasoning: The componential analysis of human abilities. Hillsdale, N.J.: Erlbaum, 1977. (b)

Sternberg, R. J. Componential investigations of human intelligence. In A. Lesgold, J. Pellegrino, S. Fokkema, & R. Glaser (Eds.), Cognitive psychology and instruction. New York: Plenum, in press.

Woodworth, R. S., & Sells, S. B. An atmosphere effect in formal syllogistic reasoning. Journal of Experimental Psychology, 1935, 18, 451-460.

Footnote

Preparation of this article was supported by Contract N0001478C0025 from the Office of Naval Research to Robert Sternberg. The research described in the article was supported by this contract, and by grant BNS76-05311 from the National Science Foundation to Robert Sternberg. Much of the research described in the article has been and is being done in collaboration with graduate students, including Morty Bernstein, Renée Elio, Michael Gardner, Marty Guyote, Jerry Ketron, Georgia Nigro, Bathsheva Rifkin, Brian Ross, Miriam Schustack, Roger Tourangeau, and Margaret Turner. I thank them all, and especially my research assistant, Barbara Conway, who has been heavily involved in many aspects of the research program. Requests for reprints should be addressed to Robert J. Sternberg, Department of Psychology, Yale University, Box 11A Yale Station, New Haven, Connecticut 06520.

Table 1

Information-Processing Components of

Transitive-Chain Theory of Syllogistic Reasoning:

Two Quantified Premises

Encoding	
ENC_{I}	Encoding of equivalence set relation
ENC_{II}	Encoding of nonequivalent symmetrical set relation
ENC_{III}	Encoding of asymmetrical set relation
Combination	
$COMB_{I-I}$	Combination of pair of equivalence set relations
$COMB_{I-II}$	Combination of equivalence with symmetrical set relation
$COMB_{II-II}$	Combination of pair of nonequivalent symmetrical set relations
$COMB_{III}$	Combination of asymmetrical set relation with any other set relation
Comparison	
CHECK	Check on combination process prior to labeling relationship between subject and predicate indeterminate
Response	
RESP	Response production

Table 2

Response-Choice Components of

Transitive-Chain Theory of Syllogistic Reasoning:

Two Quantified Premises

Combination

p_1	P(Exactly 1 pair of set relations is combined)
p_2	P(Exactly 2 pairs of set relations are combined)
p_3	P(Exactly 3 pairs of set relations are combined)
p_4	P(Exactly 4 pairs of set relations are combined)

Comparison

β_1	P(Subject chooses label that matches the atmosphere of the premises given that one label matches the atmosphere of the premises but is weaker than an alternative label)
β_2	P(Subject chooses label that matches the atmosphere of the premises and is the stronger label given that one label both matches the atmosphere of the premises and is the stronger of two alternative labels)
c	P(Subject mistakenly labels a combined representation indeterminate given that the representations generated during the combination stage include different initial components)

Table 3

Experimental Data for

Categorical and Conditional Syllogisms with Two Quantified Premises

Fits of Mathematical Models to Data

Problem Type	R^2 for Response Choice	R^2 for Response Time
Categorical		
Abstract	.97	.81
Factual	.91	---
Counterfactual	.92	---
Anomalous	.89	---
"Most-Few"	.94	---
Conditional		
Abstract	.82	---

Parameter Estimates

Response Choice Problem Type	Parameter p_1	$p_2+p_3+p_4$	β_1	β_2	c
Categorical					
Abstract	.54	.46	.81	.92	.37
Factual	.29	.71	.67	.95	.37
Counterfactual	.49	.51	.73	.94	.48
Anomalous	.47	.53	.70	.92	.48
Conditional					
Abstract	.57	.43	.76	.84	.61

Table 3 Contd.

Response Time

Problem Type: Categorical, Abstract					
ENC_I	.76	$COMB_{I-I}$	2.04	CHECK	2.54
ENC_{II}	.81	$COMB_{I-II}$	2.34	RESP	34.69
ENC_{III}	.97	$COMB_{II-II}$	2.38		
		$COMB_{III}$	2.75		

Relations of Parameters to Measured Abilities

Mean Differences

	Ability	
	Verbal	Spatial-Abstract
Categorical Syllogisms		
p_1	.05	.25*
β_1	.04	.04
β_2	.02	.03
c	.04	.04
Conditional Syllogisms		
p_1	.02	.21*
β_1	.12	.11
β_2	.03	.11
c	.14	.07

Note: Latency parameter estimates expressed in seconds.

*$p < .05$

Table 4

Information-Processing Components of

Transitive-Chain Theory of Syllogistic Reasoning:

One Quantified Premise

Encoding and Combination	
p_{1p}	of first (primary) element of first premise with second premise--no negatives in first premise
p_{1n}	of first (primary) element of first premise with second premise--one or more negatives in first premise
p_{2p}	of first (primary) element of first premise with negation of conclusion--no negatives in first premise
p_{2n}	of first (primary) element of first premise with negation of conclusion--one or more negatives in first premise
s_{1p}	of second element of first premise with second premise--no negatives in first premise
s_{1n}	of second element of first premise with second premise--one or more negatives in first premise
s_{2p}	of second element of first premise with negation of conclusion--no negatives in first premise
s_{2n}	of second element of first premise with negation of conclusion--one or more negatives in first premise
Response	
RESP	Response production

Table 5

Response-Choice Components of

Transitive-Chain Theory of Syllogistic Reasoning:

One Quantified Premise

Combination	
p_1	P(Exactly 1 pair of set relations is combined)
p_2	P(Exactly 2 pairs of set relations are combined)
t_0	P(Forming chain with negation of conclusion given that there are no negatives in first premise)
t_1	P(Forming chain with negation of conclusion given that there is one negative in first premise)
t_2	P(Forming chain with negation of conclusion given that there are two negatives in first premise)

Table 6

Experimental Data for

Categorical and Conditional Syllogisms with One Quantified Premise

Fits of Mathematical Models to Data

Problem Type	R^2 for Response Choice	R^2 for Response Time
Categorical		
Abstract	.97	.91
Conditional		
Abstract	.95	.84

Parameter Estimates

Response Choice	Parameter				
Problem Type	p_1	p_2	t_0	t_1	t_2
Categorical					
Abstract	.36	.64	.52	.48	.15
Conditional					
Abstract	.43	.57	.60	.61	.16

Response Time									
Problem Type	p_{1p}	p_{2p}	p_{1n}	p_{2n}	s_{1p}	s_{2p}	s_{1n}	s_{2n}	RESP
Categorical									
Abstract	.73	.96	.94	1.02	1.13	1.14	1.18	1.27	12.26
Conditional									
Abstract	.82	1.01	1.19	1.31	1.01	1.13	1.24	1.49	11.48

Table 6 Contd.

Relations of Parameters to Measured Abilities

Mean Differences

	Ability	
	Verbal	Spatial-Abstract
Categorical and Conditional Syllogisms Combined		
p_1	.03	.17*
t_0	.02	.23*
t_1	.06	.20*
t_2	.02	.10*

Note: Latency parameter estimates expressed in seconds.

*p <.05

Table 7

Information-Processing Components of

Mixture Theory of Transitive Inference

Linguistic Components	
PR	Premise reading
NMAR1	Decoding of unmarked adjective
MARK1	Decoding of marked adjective (as increment to NMAR1)
NCON	Establishment of congruence between question and response adjectives
QR	Question reading
Spatial Components	
NMAR2	Seriation of two terms related by unmarked adjective
MARK2	Seriation of two terms related by marked adjective (as increment to NMAR2)
NEG	Reflection of two terms related by negation (in theory as revised in light of experimental data)
PSM	Search for pivot (middle) term of array
SER	Integrating seriation of two sets of two terms
RS	Search for response in array
Neutral Component	
RES	Response production

Table 8

Experimental Data for

Linear Syllogisms

Fits of Mathematical Models to Data

Problem Type	R^2 for Response Time	R^2 for Error
Precueing: Question Last	.81	---
Precueing: Question First	.74	---
No Precueing: Adjectives Within Subjects	.84	---
No Precueing: Adjectives Between Subjects	.88	---
No Precueing: Speed Emphasis	.84	.39

Parameter Estimates

Response Time	ENC+	NEG	MARK	PSM	RS	NCON	RES+
Precueing: Question Last	4648	351	337	1136	380	---	1307
Precueing: Question First	4666	366	412	1045	695	---	836
No Precueing: Adjectives Within Subjects	2986	184	307	1154	522	538	2517
No Precueing: Adjectives Between Subjects	3124	244	380	1008	656	396	2353
No Precueing: Speed Emphasis	1354	143	327	788	485	395	1944
Response Errors (Standardized) (Speed Emphasis)	---	-.15	.38	.12	.19	.23	----

Table 8 Contd.

Relations of Parameters to Measured Abilities

(First Four Experiments)

Correlations

Response-Time Parameter	Ability	
	Verbal	Spatial-Abstract
ENC+	-.25*	-.51*
NEG	-.14	-.34*
MARK	-.20*	-.36*
PSM	-.16	-.25*
RS	-.26*	-.35*
NCON	-.31*	-.24
RES+	-.30*	-.09

Note: Parameter estimates are expressed in milliseconds. Not all elementary components could be separated. ENC+ consists of a combination of SER, PR, NMAR1, and NMAR2, with different combinations in the first two experiments from the last three. MARK consists of MARK1+MARK2-NMAR1-NMAR2. RES+ consists of a combination of RES, OR, NMAR1, NMAR2, and PR, with different combinations in different experiments.

*$p < .05$

Table 9

Information-Processing Components of

IMAJER Theory of Analogical, Classificational, Serial,

Topological, and Metaphorical Reasoning

Attribute Identification	
a	Encoding of attribute-value
Attribute Comparison	
x	Inference of relation between attribute-values
y	Mapping of higher-order relation between attribute-values
z	Application of relation between attribute-values
Attribute Checking	
t	Justification of validity of previous operations
Response	
c	Response production

Table 10

Experimental Data for

Analogies with Integral Terms

Fits of Mathematical Models to Data

Problem Type	R^2 for Response Time	R^2 for Error
People Piece (schematic picture)	.92	.59
Verbal	.86	.12
Geometric	.80	.50

Parameter Estimates

Problem Type	Parameter a	x	y	z	c	t
			Response-Time			
People Piece	.56	.13	.20	.09	.45	---
Verbal	1.29	.29	.24	.18	.41	---
Geometric	2.41	.91	1.08	.81	.43	.97
			Error (standardized)			
People Piece	----	---	.52	.48	---	---
Verbal	----	---	---	---	---	---
Geometric	.12	---	.31	.17	---	.32

Table 10 Contd.

Relations of Parameters to Measured Abilities

Correlations with Reasoning

	Parameter					
	a	x	y	z	c	t
Problem Type						
People Piece	.32	-.13	-.31	-.19	-.71*	---
Verbal	.55*	-.56*	-.54*	-.14	-.80*	---
Geometric[a]	---	---	---	.49	---	.53*

Note: Parameter estimates are expressed in seconds.

[a]Multiple correlations including error rates.

*$p < .05$

Table 11

Response-Choice Components of

Theory of Causal Inference

Inference Components	
Target Hypothesis	
PA	Weighting of positive affirming instance
NA	Weighting of negative affirming instance
PI	Weighting of positive infirming instance
NI	Weighting of negative infirming instance
Alternative Hypotheses (Distractors)	
DPA	Weighting of positive affirming evidence for two strongest distractors
DPI	Weighting of positive infirming evidence for two strongest distractors
Base Component	
B	Base weighting with no evidence

Table 12

Experimental Data for

Causal Inference Problems

Fit of Mathematical Model to Data

	R^2
Causal Inference Problems (abstract)	.86

Parameter Estimates

PA	.11	DPA	-.08
PI	-.13	DPI	.07
NA	.10		
NI	-.09	B	.38

Relations of Parameters to Measured Reasoning Ability

PA	.18	DPA	-.01
PI	-.16	DPI	.17
NA	.16		
NI	-.04	B	.04

Figure Captions

1. Hierarchical structure of componential theory of human reasoning.

2. Example of topological relations problem. The subject must choose the picture at the right that allows the dot to be placed in a position topologically equivalent to its position in the picture at the left.

3. Example of induced syllogism. The subject must induce the premises of the syllogism from the two sets of diagrams at the top, choosing for each set of diagrams a premise that is consistent with all three diagrams. Next, the subject must solve the syllogism. Then, the subject must select the set of diagrams at the bottom that is consistent with the conclusion the subject reaches, or else label the problem as "indeterminate."

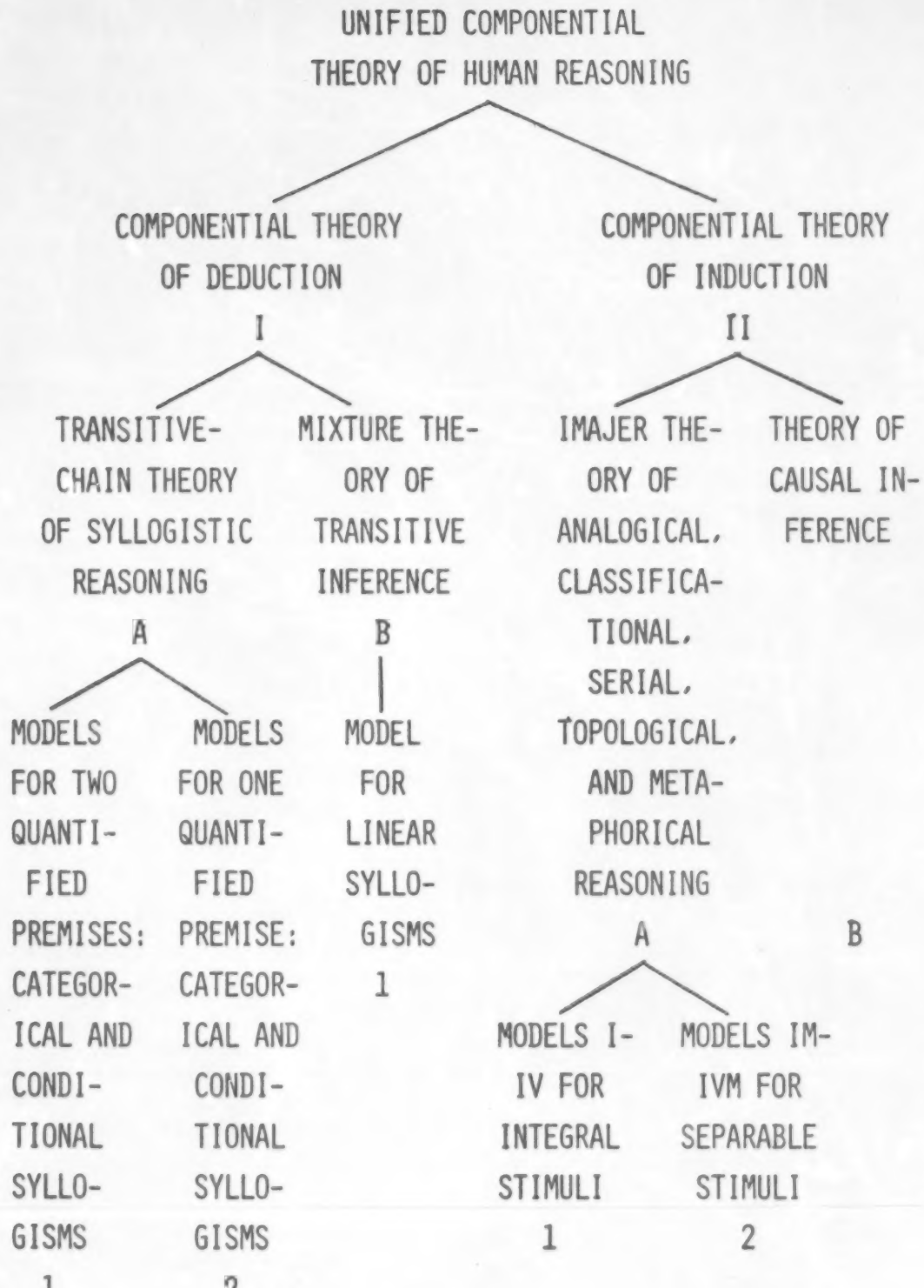

UNIFIED COMPONENTIAL THEORY OF HUMAN REASONING
COMPONENTIAL THEORY OF DEDUCTION
I
COMPONENTIAL THEORY OF INDUCTION
II
TRANSITIVE-CHAIN THEORY OF SYLLOGISTIC REASONING
A
MIXTURE THEORY OF TRANSITIVE INFERENCE
B
IMAJER THEORY OF ANALOGICAL, CLASSIFICATIONAL, SERIAL, TOPOLOGICAL, AND METAPHORICAL REASONING
A
THEORY OF CAUSAL INFERENCE
B
MODELS FOR TWO QUANTIFIED PREMISES: CATEGORICAL AND CONDITIONAL SYLLOGISMS
1
MODELS FOR ONE QUANTIFIED PREMISE: CATEGORICAL AND CONDITIONAL SYLLOGISMS
2
MODEL FOR LINEAR SYLLOGISMS
1
MODELS I-IV FOR INTEGRAL STIMULI
1
MODELS IM-IVM FOR SEPARABLE STIMULI
2

Figure 1

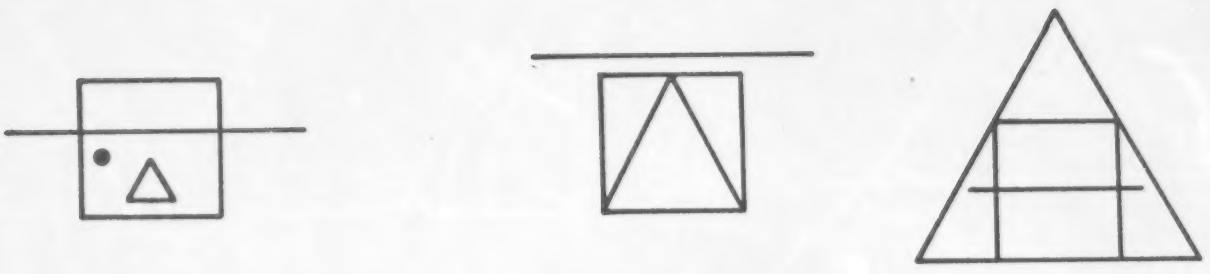
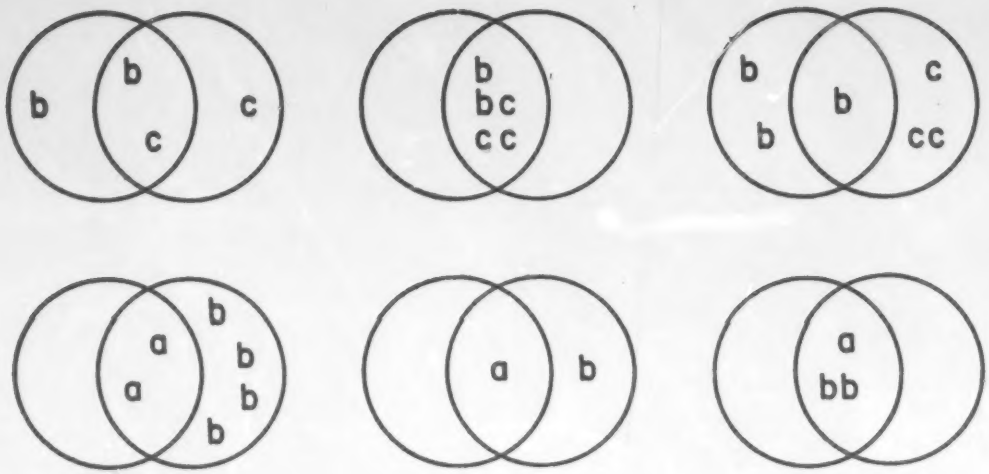


Figure 2.

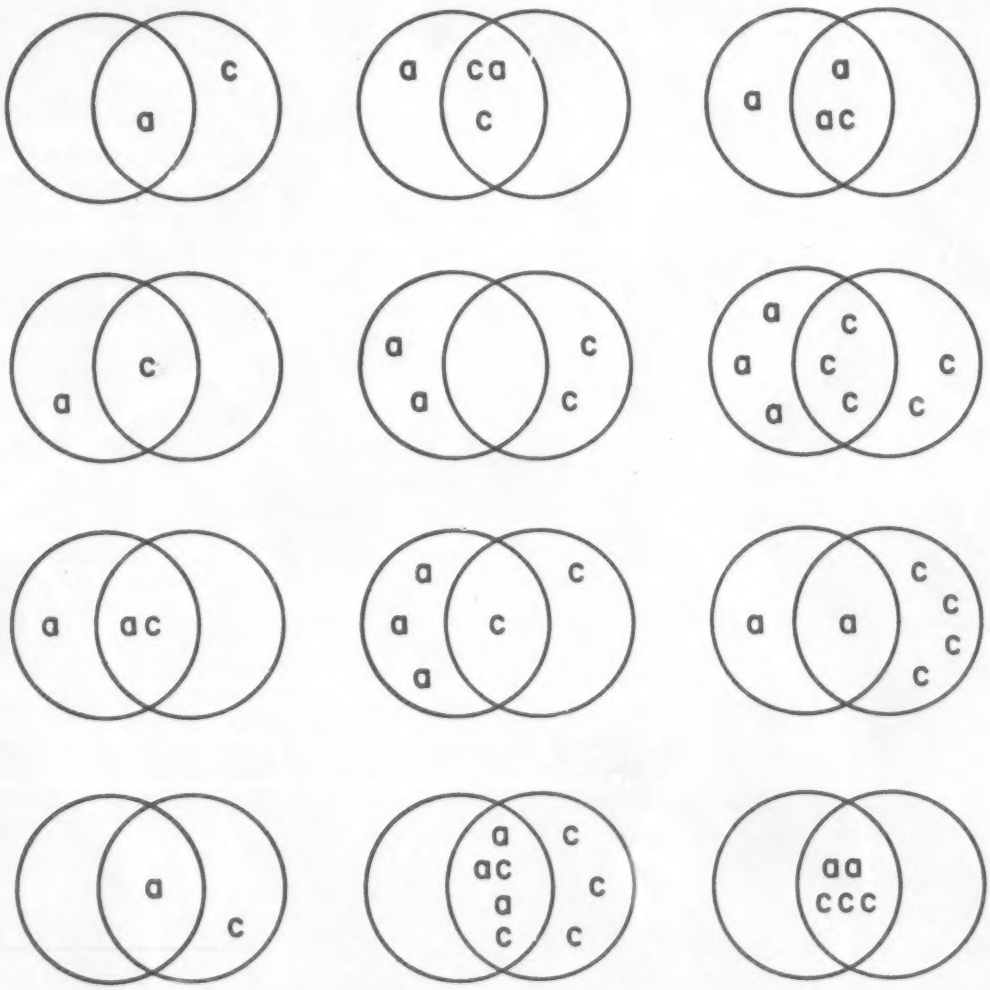


(A)

(B)

(C)

(D)



(E) Indeterminate

Figure 3